**6. Introduction:** concentrating collectors work by interposing an optical device between the source of radiation and the energy-absorbing surface. Therefore, for concentrating collectors, both optical and thermal analyses are required. The concentration ratio (C) is defined as the ratio of the aperture area to the receiver-absorber area; that is,

$$C = (A_a/A_r) \dots (53)$$

For flat-plate collectors with no reflectors, (**C = 1**). For concentrators, **(C)** is always greater than 1.

## 6.1. Optical Analysis of Parabolic Trough Collectors (PTCs):

A cross-section of a parabolic trough collector is shown in Figure (7), where various important factors are shown. The incident radiation on the reflector at the rim of the collector (where the mirror radius, $r_r$, is maximum) makes an angle, $\varphi_r$ , with the center line of the collector, which is called the ***rim angle*** . The equation of the parabola in terms of the coordinate system is

$$\boldsymbol{y^2 = 4fx \dots (54)}$$ $\boldsymbol{f}$ = Parabola focal distance (m).

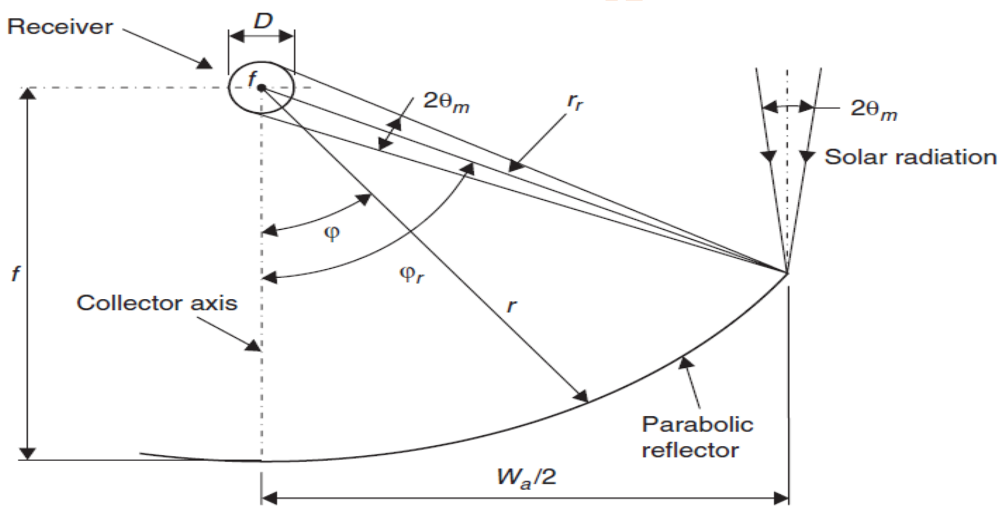


Figure (7): Cross-section of a parabolic trough collector with circular receiver.

The size of the receiver (diameter D) required to intercept all the solar image can be obtained from trigonometry and Figure (7), given by

$$\boldsymbol{D = 2r_r \, sin(\theta_m)} \dots (55)$$ $\boldsymbol{\theta_m}$ = half acceptance angle (degrees).

For a parabolic reflector, the radius, $r$, shown in Figure (1) is given by

$$r = (2f/1 + \cos(\varphi)) \dots (56)$$

$\varphi$ = Angle between the collector axis and a reflected beam at the focus.

At the rim angle, $\varphi_r$, becomes

$$\boldsymbol{r_r} = (2f/1+\cos(\varphi_r)) \quad \dots (57)$$

Another important parameter related to the rim angle is the aperture of the parabola, $\boldsymbol{W_a}$. From Figure (7) and simple trigonometry, it can be found that

$$W_a = 2r_r \sin(\varphi_r) \dots (58)$$

$$W_a = [4f\sin(\varphi_r)/(1+\cos(\varphi_r))] \dots (59)$$

$$W_a = 4f\tan[\varphi_r/2] \dots (60)$$

For a tubular receiver→ $C = (W_a/\pi D) \dots (61)$

By replacing D and $W_a$ with equations (55) and (58), respectively, we get

$$C = (\sin(\varphi_r)/\pi\sin(\theta_m)) \dots (62)$$

when $\varphi_r = 90°$ and $\sin\varphi_r = 1 \rightarrow C_{max} = (1/\pi\sin(\theta_m)) \dots (63)$

The curve length of the reflective surface is given by

$$S = \frac{H_p}{2}\left\{\sec\left[\frac{\varphi_r}{2}\right]\tan\left[\frac{\varphi_r}{2}\right] + ln\left\langle\sec\left[\frac{\varphi_r}{2}\right] + \tan\left[\frac{\varphi_r}{2}\right]\right\rangle\right\} \dots (64)$$

$H_p$ = Lactus rectum of the parabola (m).

**<u>Example</u>**: For a parabolic trough collector with a rim angle of 70°, aperture of 5.6 m, and receiver diameter of 50 mm, estimate the focal distance, the concentration ratio, the rim radius, and the length of the parabolic surface.
**<u>Solution</u>**:

$$W_a = 4f\tan[\varphi_r/2] \rightarrow f = W_a/4\tan[\varphi_r/2] = 5.6/4\tan[35] = 2m$$

$$C = (W_a/\pi D) = 5.6/0.05\pi = 35.7$$

$$\boldsymbol{r_r} = \frac{2f}{1+cos(\varphi_r)} = \frac{2\times 2}{1+cos(70)} = 2.98\,m$$

The parabola lactus rectum, $H_p$ , is equal to $W_a$ at $\varphi_r = 90^{\circ}$

$$H_p = W_a = 4f\tan[\varphi_r/2] = 4\times 2\times\tan(90/2) = 8\,m$$

$$S = \frac{H_p}{2}\left\{\sec\left[\frac{\varphi_r}{2}\right]\tan\left[\frac{\varphi_r}{2}\right] + ln\left\langle\sec\left[\frac{\varphi_r}{2}\right] + \tan\left[\frac{\varphi_r}{2}\right]\right\rangle\right\}$$
$$= \frac{8}{2}\{\sec[35]\tan[35] + ln\langle\sec[35] + \tan[35]\rangle\} = 6.03\ m$$

## 6.2. Thermal Analysis of Parabolic Trough Collectors:

The generalized thermal analysis of a concentrating solar collector is similar to that of a flat-plate collector. Two such designs are available, parabolic trough collector with a ***bare*** tube and the ***glazed tube*** receiver. In both cases, the calculations must include radiation, conduction, and convection losses.
For a bare tube receiver and assuming no temperature gradients along the receiver, the loss coefficient considering convection and radiation from the surface and conduction through the support structure is given by

$$U_L = h_w + h_r + h_c \quad \ldots (65)$$

$$\text{radiation coefficient},\ h_r = 4\,\sigma\,\varepsilon\,T_r^3 \ \ldots (66)$$

For the wind loss coefficient, the Nusselt number can be used.

$$N_u = 0.4 + 0.54\,(Re)^{0.52} \ \ldots (67) \qquad 0.1 < Re < 1000$$

$$N_u = 0.3\,(Re)^{0.6} \ \ldots (68) \qquad 1000 < Re < 50{,}000$$

Estimation of the conduction losses requires knowledge of the construction of the collector, i.e., the way the receiver is supported.
Usually, to reduce the heat losses, a concentric glass tube is employed around the receiver. The space between the receiver and the glass is usually evacuated, in which case the convection losses are negligible. In this case, $U_L$, based on the receiver area $A_r$, is given by

$$U_L = \left[\frac{A_r}{(h_w + h_{r,c-a})A_c} + \frac{1}{h_{r,r-c}}\right]^{-1} \ldots (69)$$

$h_{r,c-a}$ = Linearized radiation coefficient from cover to ambient (W/m$^2$-K).
$A_c$ = External area of glass cover (m$^2$).
$h_{r,r-c}$ = Linearized radiation coefficient from receiver to cover, given by

$$h_{r,r-c} = \frac{\sigma(T_r^2 + T_c^2)(T_r + T_c)}{\frac{1}{\varepsilon_r} + \frac{A_r}{A_c}\left(\frac{1}{\varepsilon_c} - 1\right)} \ldots (70)$$

To estimate the glass cover properties, the temperature of the glass cover, $T_c$, is required. This temperature is closer to the ambient temperature than the receiver temperature.

Therefore, by ignoring the radiation absorbed by the cover, $T_c$ may be obtained from an energy balance:

$$A_c(h_{r,c-a} + h_w)(T_c - T_a) = A_r h_{r,r-c}(T_r - T_c) \dots (71)$$

$$T_c = \frac{A_r h_{r,r-c} T_r + A_c(h_{r,c-a} + h_w) T_a}{A_r h_{r,r-c} + A_c(h_{r,c-a} + h_w)} \dots (72)$$

The procedure to find $T_c$ is by iteration, i.e., estimate $U_L$ from equation (69) by considering a random $T_c$ (close to $T_a$). Then, if $T_c$ obtained from equation (72) differs from original value, iterate. Usually, no more than two iterations are required.

$$\text{overall heat transfer coeff.} \rightarrow U_o = \left[\frac{1}{U_L} + \frac{D_o}{h_{fi} D_i} + \frac{D_o \ln(D_o/D_i)}{2k}\right]^{-1} \dots (73)$$

$D_o, D_i$ = Receiver outside, and inside tube diameter (m).
$h_{fi}$ = Convective heat transfer coefficient inside the receiver tube (W/m$^2$ -K).

The convective heat transfer coefficient, $h_{fi}$, can be obtained from the standard pipe flow equation:

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.4} \dots (74)$$

It should be noted that equation (74) is for turbulent flow (Re > 2300). For laminar flow, Nu = 4.364 = constant.

$Re$ = Reynolds number = $(\rho V D_i/\mu)$.

$Pr$ = Prandtl number = $(c_p \mu/k_f)$.

$\mu$ = Fluid viscosity (kg/m-s).

$k_f$ = Thermal conductivity of fluid (W/m-K).

$$\text{useful energy delivered from } PTC, \rightarrow Q_u = I_B \eta_o A_a - A_r U_L (T_r - T_a) \dots (75)$$

Note that, because concentrating collectors can utilize only beam radiation, $I_B$ is used instead of the total radiation $I_t$.
The useful energy gain per unit of collector length can be expressed in terms of the local receiver temperature, $T_r$ , as

$$q'_u = (Q_u/L) = (I_B \eta_o A_a/L) - (A_r U_L (T_r - T_a)/L) \dots (76)$$

In terms of the energy transfer to the fluid at the local fluid temperature, $T_f$

$$q'_u = \frac{(A_r/L)(T_r - T_f)}{\frac{D_o}{h_{fi} D_i} + \frac{D_o \ln(D_o/D_i)}{2k}} \dots (77)$$

If $T_r$ is eliminated from equations (76) and (77), we have

$$q'_u = F' \frac{A_a}{L}\left[I_B\eta_o - \frac{U_L}{C}\left(T_f - T_a\right)\right] \ldots (78)$$

$$\text{collector efficiency factor,} \rightarrow F' = \frac{(1/U_L)}{\frac{1}{U_L}+\frac{D_o}{h_{fi}D_i}+\frac{D_o\ln(D_o/D_i)}{2k}} = \frac{U_o}{U_L} \ldots (79)$$

As for the flat-plate collector, $T_r$ in equation (75) can be replaced by $T_i$ by the heat removal factor, and equation (75) can be written as

$$Q_u = F_R[I_B\eta_o A_a - A_r U_L(T_i - T_a)] \ldots (80)$$

The collector efficiency can be obtained by dividing $Q_u$ by $(I_B A_a)$. Therefore,

$$\eta = F_R\left[\eta_o - U_L\left(\frac{T_i - T_a}{I_B C}\right)\right] \ldots (81),$$

Where $\rightarrow C = (A_a/A_r)$

**Example**: A 20 m long parabolic trough collector with an aperture width of 3.5 m has a pipe receiver of 50 mm outside diameter and 40 mm inside diameter and a glass cover of 90 mm in diameter. If the space between the receiver and the glass cover is evacuated, estimate the overall collector heat loss coefficient, the useful energy gain, and the exit fluid temperature. The following data are given:

Absorbed solar radiation = 500 W/m$^2$.
Receiver temperature =260 °C = 533 K.
Receiver emissivity, $\boldsymbol{\varepsilon_r}$ = 0.92.
Glass cover emissivity, $\boldsymbol{\varepsilon_g}$ = 0.87.
Circulating fluid, $\boldsymbol{c_p}$ = 1350 J/kg-K.
Entering fluid temperature =220°C = 493 K.
Mass flow rate = 0.32 kg/s.
Heat transfer coefficient inside the pipe = 330 W/m$^2$ -K.
Tube thermal conductivity k = 15 W/m-K.
Ambient temperature = 25°C = 298 K.
Wind velocity = 5 m/s.

**Solution**:

$$receiver\ area, A_r = \pi D_o L = \pi \times 0.05 \times 20 = 3.14\ m^2$$
$$glass\ cover\ area, A_g = \pi D_g L = \pi \times 0.09 \times 20 = 5.65\ m^2$$
$$unshaded\ aperture\ area, A_a = (3.5 - 0.09) \times 20 = 68.2\ m^2$$

Next, a glass cover temperature, $T_g$ , is assumed in order to evaluate the convection and radiation heat transfer from the glass cover. This is assumed equal to 64°C =337 K. The actual glass cover temperature is obtained by iteration by neglecting the interactions with the reflector. The convective (wind) heat transfer coefficient $h_{c,c-a} = h_w$ of the glass cover can be calculated from equation (21). First, the Reynolds number needs to be estimated at the mean temperature (1/2) (25 + 64) = 44.5°C.

$$\rho = 1.11\ kg/m^3 :\ \mu = 2.02 \times 10^{-5}\ kg/m.s : k = 0.0276\ W/m.K$$

$$Re = (\rho V D_g/\mu) = (1.11 \times 5 \times 0.09/2.02 \times 10^{-5}) = 24{,}728$$

$$N_u = 0.3(Re)^{0.6} = 129.73$$

$$h_{c,c-a} = h_w = (N_u k/D_g) = (129.73 \times 0.0276/0.09) = 39.8\ W/m^2.K$$

$$h_{r,c-a} = \varepsilon_g \sigma (T_g + T_a)(T_g^2 + T_a^2)$$
$$= 0.87 \times (5.67 \times 10^{-8})(337 + 298)(337^2 + 298^2) = 6.34\ W/m^2.K$$

$$h_{r,r-c} = \frac{\sigma(T_r^2 + T_g^2)(T_r + T_g)}{(1/\varepsilon_r) + (A_r/A_g)\langle(1/\varepsilon_g) - 1\rangle}$$
$$= \frac{(5.67 \times 10^{-8})(533^2 + 337^2)(533 + 337)}{(1/0.92) + (0.05/0.09)\langle(1/0.87) - 1\rangle} = 16.77\ W/m^2.K$$

$$U_L = \left[\frac{A_r}{(h_w + h_{r,c-a})A_g} + \frac{1}{h_{r,r-c}}\right]^{-1} = \left[\frac{0.05}{(39.8 + 6.34) \times 0.09} + \frac{1}{16.77}\right]^{-1}$$
$$= 13.95\ W/m^2.K$$

Since $U_L$ is based on the assumed $T_g$ value, we need to check if the assumption made was correct. Using Eq. (72), we get

$$T_g = \frac{A_r h_{r,r-c} T_r + A_c(h_{r,c-a} + h_w)T_a}{A_r h_{r,r-c} + A_c(h_{r,c-a} + h_w)} = \frac{(3.14 \times 16.77 \times 260) + 5.65(6.34 + 39.8)25}{(3.14 \times 16.77) + 5.65(6.34 + 39.8)} = 64.49\ °C$$

$$F' = \frac{(1/U_L)}{\frac{1}{U_L} + \frac{D_o}{h_{fi} D_i} + \frac{D_o \ln(D_o/D_i)}{2k}} = \frac{(1/13.95)}{\frac{1}{13.95} + \frac{0.05}{330 \times 0.04} + \frac{0.05 \ln(0.05/0.04)}{2 \times 15}}$$
$$= 0.945$$

$$F_R = \frac{m^{\circ} c_p}{A_r U_L}\left[1 - exp\left(\frac{-U_L\ A_r\ F'}{m^{\circ} c_p}\right)\right]$$
$$= \frac{0.32 \times 1350}{3.13 \times 13.95}\left[1 - exp\left(-\frac{13.95 \times 0.95 \times 3.14}{0.32 \times 1350}\right)\right] = 0.901$$

$$Q_u = F_R[SA_a - A_r U_L(T_i - T_a)] = 0.901[500 \times 68.2 - 3.14 \times 13.95(220 - 25)]$$
$$= 23{,}031\ W$$

$$Q_u = m^{\circ} c_p\ (T_o - T_i) \rightarrow\ T_o = T_i + (Q_u/m^{\circ} c_p) = 220 + (23{,}031/0.32 \times 1350)$$
$$= 273.3\ °C$$